ALGEBRA I Mid-Term Examination 19 September 2006

Instructions: Some questions are simpler than others. Therefore, read the entire question paper carefully before attempting to answer. All questions carry equal marks.

- 1. State and prove Lagrange's theorem.
- 2. Determine the order sequence of a cyclic group of order 20.
- 3. Classify all groups that have at most three conjugacy classes.
- 4. Prove that there exists a unique group of order 15.
- 5. Let N be the normaliser of a Sylow subgroup of a finite group G. Prove that N is normal in G if and only if N = G.
- 6. Prove that no group of order 30 is simple.